

## Magnetically Induced Chessboard Pattern in the Conductance of a Kondo Quantum Dot

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We quantitatively describe the main features of the magnetically induced conductance modulation of a Kondo quantum dot—or chessboard pattern—in terms of a constant-interaction double quantum dot model. We show that the analogy with a double dot holds down to remarkably low magnetic fields. The analysis is extended by full 3D spin density functional calculations. Introducing an effective Kondo coupling parameter, the chessboard pattern is self-consistently computed as a function of magnetic field and electron number, which enables us to explain our experimental data quantitatively.

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A quantum dot [1] with a finite net electron spin strongly coupled to its leads can exhibit the Kondo effect [2,3]. The Kondo effect in quantum dots manifests itself as an enhanced conductance in the Coulomb blockade regime and occurs for temperatures and source-drain voltages below an energy scale set by the Kondo temperature [4–6]. Assuming continuous filling of spin-degenerate single-particle levels, the dot is expected to have either total electron spin  $S = 0$  (for electron number  $N$  even) or  $S = 1/2$  (for  $N$  odd). The Kondo effect is therefore expected only for odd  $N$ , hence giving rise to an “even-odd effect” in the Coulomb valley conductance. A wide range of experiments, however, has shown a clear deviation from this picture [7–16]. Particularly striking is the observation of a “chessboard pattern” in the dot conductance as a function of magnetic field  $B$  and gate voltage  $V_g$  [11,12,14–16]. Characteristic for this pattern is the alternation of high and low valley conductance regions as a function of  $B$  within the same Coulomb valley, i.e., for constant  $N$ . In addition, the conductance also alternates when  $N$  is changed by sweeping  $V_g$  at constant  $B$ . The distinct regions in the  $V_g, B$  plane of either high or low conductance are associated with the fields of a chessboard due to the similar appearance when the conductance is plotted in color scale [see Fig. 1(b)]. It has been experimentally shown that the enhanced conductance in certain Coulomb blockade regions can be ascribed to the Kondo effect, for both  $N$  odd and even [11,12,14–16].

In this Letter, we start by presenting our experimental data on a single lateral quantum dot clearly exhibiting the chessboard pattern. Next, we calculate the skeleton of the chessboard pattern with a constant-interaction (CI) model of two capacitively interacting dots, formed by the two lowest Landau levels (LLs)—an analogy (partly qualitatively) applied before at high magnetic fields [12,17]—and obtain the characteristic hexagonal, double dot (DD) stability diagram [18]. Interestingly, in this work we find

that, due to small  $N$  and the shallow potential, higher LLs start to fill only below a few tenths of a tesla. Thus the DD analogy applies down to remarkably low magnetic fields. We finally perform full, 3D spin density functional (SDF) calculations for our device and introduce an effective Kondo coupling, derived entirely from the self-consistent results. Hence we can simultaneously calculate the electronic states of the dot and an estimate of the Kondo coupling, exhibiting the chessboard structure, and quantitatively explain some of its subtler features.

Our quantum dot is shown in Fig. 1(a). Metal gates are deposited on top of a GaAs/AlGaAs heterostructure with a two-dimensional electron gas (2DEG) 100 nm below the surface [18]. By depleting the 2DEG below the gates, the quantum dot is defined. Current can flow from the source (S) to the drain (D) contact. The electron number is varied by sweeping the left gate voltage  $V_{gl}$ . The SDF calculations show that our dot typically contains  $\sim 20$ –40 electrons.

Figure 1(b) shows a color scale plot of the linear conductance  $G$  through the dot as a function of  $B$  and  $V_{gl}$ . Red (blue) corresponds to large (small)  $G$  [see scale in Fig. 1(b)]. For the most negative values of  $V_{gl}$  the coupling of the dot to the leads is weak. This results in relatively sharp Coulomb peaks (red lines) and low valley conductance (dark blue regions). However, if  $V_{gl}$  is increased, the valley conductance reaches considerable values ( $\sim e^2/h$ ) in certain regions of the  $(B, V_{gl})$  plane. Most strikingly, the regions of low and high valley conductance alternate both along the  $V_{gl}$  and the  $B$  axis in a regular fashion, resulting in the aforementioned chessboard pattern. The  $V_{gl}$  period ( $\sim 10$  mV) is set by the energy required for adding an extra electron to the dot (addition energy), whereas the  $B$  period ( $\sim 0.1$  T) corresponds to adding a flux quantum to the effective dot area. Based on the temperature dependence of  $G$  in the high valley conductance regions (not shown here), we can

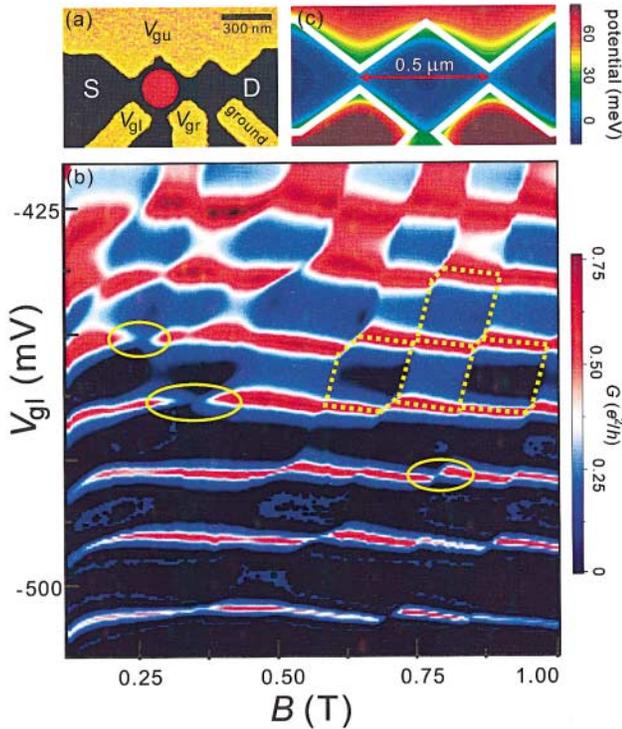


FIG. 1 (color). (a) Scanning electron micrograph of the device. Metal gates are yellow; dot is indicated by a red circle. Ungated 2DEG mobility is  $2.3 \times 10^6 \text{ cm}^2/(\text{Vs})$  and electron density is  $1.9 \times 10^{15} \text{ m}^{-2}$  at 4.2 K. Nominal dot size is  $320 \times 320 \text{ nm}^2$ . (b) Color scale plot of the experimental linear conductance  $G$  through dot as function of  $B$  and  $V_{gl}$  at 10 mK. The dotted hexagons highlight the shape of a few chessboard fields; ellipses indicate some regions where Coulomb peak suppression occurs. (c) Calculated self-consistent potential landscape of device; white lines denote contours of the metal gates.

ascribe the enhancement of  $G$  to the Kondo effect. The transition from low to high valley conductance is associated with an abrupt jump of the  $V_{gl}$  position of the Coulomb peaks. In some cases [see ellipses in Fig. 1(b)] the jump is accompanied by a suppression of the peak height. Below we present a quantitative description for the experimental features discussed above, using first an intuitive, though quantitative, approach followed by a fully self-consistent simulation of our device.

In the region where the two lowest LLs, labeled LL0 and LL1, are occupied, the outline of the chessboard pattern can be considered to be a DD [Fig. 2(a)] stability diagram, where now  $B$  acts as one of the gates.  $B$  couples to both “dots” (i.e., LLs) but with different “lever arms.” Similarly,  $V_{gl}$  couples with different capacitances to the two LLs. We write an elementary CI functional for the total energy  $E$ ,

$$E(N_0, N_1) = \sum_{nm} \sum_{\sigma} \varepsilon_{nm} + \frac{1}{2D} (C_{11}\rho_0^2 + C_{00}\rho_1^2) - \frac{C_{01}}{D} \rho_0\rho_1, \quad (1)$$

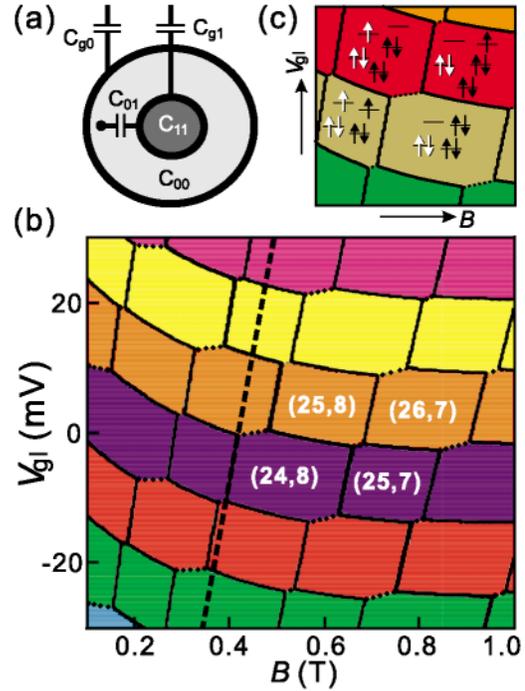


FIG. 2 (color). (a) Schematic of the dot in terms of two Landau levels (LLs). (b) Results of numerical minimization of Eq. (1). Color stripes are regions of constant  $N = N_0 + N_1$ . Numbers in parentheses show  $N_0, N_1$ . Solid lines bound regions of constant  $N_0$ . Dotted lines show “ $N_1$  boundaries”. The  $N_1$  boundary lengths alternate [compare to Fig. 1(b)]. Capacitances and dot parabolicity estimated for  $N_0^0 = 25$ ,  $N_1^0 = 10$ ,  $B_0 = 0.6 \text{ T}$ :  $C_{00} = 3.3$ ,  $C_{11} = 2.2$ ,  $C_{01} = -2.1$ ,  $C_{g0} = -0.10$ , and  $C_{g1} = -0.056$  (all in attofarad),  $\omega_0 = 0.4 \text{ meV}$ . The dashed line is the approximate third LL filling boundary. (c) Schematic honeycomb structure in the  $B, V_{gl}$  plane resulting from the double dot model. Typical occupancy configurations (energy versus position) of LL1 (white-on-black arrows) and LL0 (black arrows) at the dot boundary are shown for different honeycomb cells.

where  $\rho_{0(1)} \equiv (N_{0(1)}e - C_{g0(1)}V_{gl})$ ,  $N_{0(1)}$  is the number of electrons in LL0(1),  $\varepsilon_{nm} \equiv (2n + |m| + 1)\hbar\tilde{\omega} + m\hbar\omega_c$  are the Fock-Darwin (FD) energy levels [19], and the sum is over the lowest two LLs. A LL consists of all  $n, m$  that satisfy LL index  $\lambda = n + \frac{1}{2}(|m| + m) = \text{const}$  (i.e., 0 or 1 for our specific case), and  $\sigma$  is the spin index (note that at the magnetic fields considered here the Zeeman splitting is negligible, so we assume spin-degenerate states). Here,  $\tilde{\omega}^2 \equiv \omega_0^2 + \omega_c^2$  with  $\omega_0$  the bare confining frequency and  $\omega_c$  the cyclotron frequency.  $C_{ij}$  are the capacitance matrix elements (off-diagonal elements are always negative [18,20]),  $D \equiv C_{00}C_{11} - C_{01}^2$ . Here  $V_{gl}$  is defined relative to the gate voltage that induces  $N_{0(1)}^0$  electrons [i.e.,  $N_{0(1)}^0$  is the number of electrons in LL0(1) at  $V_{gl} = 0$ ].

First, we estimate capacitances  $C_{ij}$  and bare confining potential  $\omega_0$  from the full SDF calculations for  $(N_0^0, N_1^0)$  electrons and for a “central” magnetic field  $B_0$  (see

caption of Fig. 2) [21]. Then, we numerically minimize Eq. (1) with respect to  $N_0$  and  $N_1$ , keeping  $C_{ij}$  and  $\omega_0$  constant [even though in principle these change slightly over our  $(B, V_{gl})$  range]. We find a “honeycomb” structure characteristic for DDs [18]. As shown in Figs. 2(b) and 2(c), this structure clearly emerges and compares reasonably well with the experimental data of Fig. 1(b).

Figure 2(c) schematically shows the population of the lowest two LLs in four neighboring honeycomb cells.  $B$  principally induces depopulation of LL1 to LL0;  $V_{gl}$  changes the total electron number  $N = N_0 + N_1$ . Note that the dotted boundaries between  $N_1$  and  $N_1 + 1$  (“ $N_1$  borders”) are considerably shorter than the solid boundaries between  $N_0$  and  $N_0 + 1$ . Weak tunnel coupling of the inner LL (LL1) to the leads causes these  $N_1$  borders to appear as gaps (or small offsets) in the Coulomb oscillations [22], which are clearly seen in Fig. 1(b). Analysis of Eq. (1) shows that the length of the  $N_1$  borders, in both  $B$  and  $V_{gl}$ , is proportional to  $C_{11} - |C_{01}|$ . Therefore, the short  $N_1$  borders indicate that LL0 strongly screens (via  $C_{01}$ ) the inner LL1. In other words, most of the self-capacitance of LL1 is taken up by capacitance to LL0. This screening of the inner LL is ultimately what produces the nearly rectangular chessboard pattern of the Kondo conductance (see below). Furthermore, the length of the  $N_1$  borders in Figs. 2(b) and 2(c) alternates, as observed in the experimental data [Fig. 1(b)]. The shorter  $N_1$  borders appear as only small discontinuities in the experimental Coulomb oscillations. From Eq. (1) it follows that this alternating pattern results from the spin degeneracy of the FD levels. Specifically, increasing  $B$  and transferring a LL1 electron to LL0, at fixed  $N$ , costs an additional LL0 level spacing when  $N_0$  changes from even to odd. The main feature of the experimental data that is not present in Figs. 2(b) and 2(c) is the increased valley conductance due to the Kondo effect. We show below, using our SDF analysis, that this increased valley conductance appears in the hexagons where  $N_0$  is odd.

Our SDF calculations for realistic, 3D, lateral semiconductor quantum dot structures have been thoroughly described before [23]. We compute self-consistent eigenvalues  $\varepsilon_{p\sigma}$ , eigenfunctions  $\psi_{p\sigma}$ , occupancies  $n_{p\sigma}$ , and tunneling coefficients  $\gamma_{p\sigma}$ , with  $p$  the orbital and  $\sigma$  the spin indices, as well as the total interacting energy  $F$  of the dot-gate-leads system, all as a function of  $N$ ,  $V_{gl}$ , and  $B$ . However, we cannot compute the coherent Kondo-assisted conductance of the dot from the ground state properties provided by the SDF calculation. Instead, we introduce an effective Kondo parameter, which enables us to reproduce the conductance modulation characteristic for the chessboard pattern.

In Ref. [12] it was qualitatively argued that the alternating Kondo conductance with varying  $B$  at fixed  $N$  observed in their experiments in the strong edge state regime resulted from Coulomb regulated redistribution of electrons, one at a time, from LL1 to LL0. Since LL0 was

assumed much more strongly coupled to the leads, the Kondo effect was argued to occur only when  $N_0$  is odd. Here, the computed electronic structure allows us to *quantitatively* exhibit this depopulation process, to show how it varies in phase from one Coulomb valley to the next and to show how the  $B$ -dependent tunneling coefficients affect the structure of individual chessboard fields. The model’s assumption of spin-degenerate states is accounted for at the end of the paper.

In general, a dot can contain many singly occupied electron states, for which spin-flip can occur via cotunneling to the leads. Such spin-flip processes provide for the effective off-diagonal coupling in the  $s$ - $d$  model, which is the basis of all Kondo physics [2]. To estimate the contribution of all such processes to the Kondo effect, we define a “Kondo parameter”  $K$  as the sum of all cotunneling amplitudes that leave the ground state unchanged except for the flip of a single spin

$$K \equiv \sum_{p,\sigma} n_{p,\sigma} (1 - n_{p,\bar{\sigma}}) \sqrt{\gamma_{p,\sigma} \gamma_{p,\bar{\sigma}}} \left( \frac{1}{E_C^N} - \frac{1}{E_C^{N-1}} \right), \quad (2)$$

where  $E_C^N \equiv F(N+1, V_{gl}, B) - F(N, V_{gl}, B)$  and  $\bar{\sigma}$  is the spin opposite to  $\sigma$ . Signatures of spin polarization in the leads [22] are weakly visible for the most negative values of  $V_{gl}$  in Fig. 1(b), but are negligible in the Kondo regime.

A color scale plot of  $K$  in the  $(B, V_{gl})$  plane is shown in Fig. 3, for  $N = 32$ – $37$ . Here, the  $V_{gl}$  dependence has been approximated as follows. The two denominators in Eq. (2) are addition energies, which are, to a good approximation, linear in  $V_{gl}$  and vanish at the Coulomb oscillations.

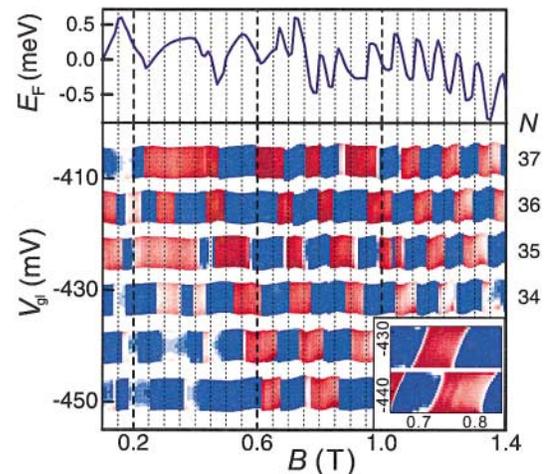


FIG. 3 (color). Color scale plot of Kondo parameter  $K$  in the  $B, V_{gl}$  plane. Red (blue) corresponds to large (small)  $K$ . Third LL filling begins below  $\sim 0.5$  T.  $K$  is calculated in the Coulomb valley center at each  $B$  (see text). Upper panel shows Fermi energy vs  $B$  for  $N = 37$  and fixed  $V_{gl} = -407$  mV.  $E_F$  drops at each reconstruction (i.e., at each end of each Kondo zone). Inset:  $K$  fully calculated from Eq. (2) on fine mesh for small  $B, V_{gl}$  region (i.e., no approximation for the  $V_{gl}$  dependence is used here).

Thus,  $E_C^N \approx (e^2/2C) + e\alpha(V_{gl} - V_{gl}^{N,\min}) + \varepsilon_{N+1}$ , where  $(e^2/2C) \equiv (\partial^2 F/\partial N^2)$  defines  $C$ , and  $\partial F(N, V_{gl}^{N,\min}, B)/\partial N = 0$  defines the valley center gate voltage  $V_{gl}^{N,\min}$ , and where  $\alpha = C_{\text{dot-gate}}/C$  is the lever arm (a similar analysis holds for  $E_C^{N-1}$ ). By calculating the full electronic structure only near the valley centers, we can determine the CI parameters  $V_{gl}^{N,\min}$ ,  $C$ , and  $\alpha$  and thereby show the  $V_{gl}$  dependence of  $K$  due to the addition energies, Fig. 3. The full calculation of Eq. (2) on a mesh of  $B$ ,  $V_{gl}$  values, which is numerically taxing, is shown for a small region in the inset of Fig. 3.

The alternating pattern of Kondo “zones” in successive Coulomb valleys is evident. Even in this low magnetic field regime, the coupling of the LL0 states to the leads typically exceeds that of the LL1 states by 2 orders of magnitude. Therefore, although the parameter  $K$  is a sum of all possible cotunneling amplitudes, the amplitudes of the LL0 states dominate. Hence, even when  $N$  is odd,  $K$  is negligible as long as  $N_0$  is even.

Within a Kondo zone an abrupt increase of  $K$  is followed by a gradual decrease. This results from the contraction, with  $B$ , of the half-filled orbit at the dot edge, and the resulting decrease of its tunnel coefficient. When another electron depopulates from LL1 to that orbit, that spin-flip process is no longer available, and  $K$  collapses. Depopulation of LL1 coincides with a drop of the dot’s Fermi level [24] relative to the leads (Fig. 3, top panel). The denominators in Eq. (2) are responsible for the increase of  $K$  away from the valley centers; a feature which is clearly observed in the experiment Fig. 1(b).

The approximation used in Fig. 3 includes the  $V_{gl}$  dependence of the charging energies, but not that of the  $n_{p,\sigma}$ . The latter dependence, specifically that of the transfer from LL1 to LL0, controls the slant of the chessboard fields. This slant emerges only in the calculation of  $K$  without approximation, as shown for a small region of  $V_{gl}$  and  $B$  in the inset of Fig. 3.

Finally, we comment on our assumption of spin-degenerate states. For  $N$  odd, this requires only that the Zeeman splitting ( $\sim 10 \mu\text{eV}$ ) is negligible, which is the case in the  $B$  range considered. However, for  $N$  even, a singly filled orbital in LL0 implies another singly filled orbital in LL1 and these electrons have, in general, an exchange interaction. In Ref. [25] it was found that this exchange interaction diminishes rapidly with  $N$ . For our dot, we explicitly determine the singlet-triplet splitting for various (even)  $N$  and  $B$  by computing separate ground states constrained to  $S = 0$  and  $S = 1$ . We find that the splitting is typically tens of  $\mu\text{eV}$ . The experimental signature of a split ground state is a split Kondo resonance

(in source-drain voltage). Our analysis suggests that this splitting, which has been observed [9,15], would be characteristic of  $N$  even in regimes where Zeeman energy is small.

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