Linear and non-linear transport through coupled quantum dots

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Abstract

We have measured linear and non-linear transport through a double quantum-dot system fabricated from the two-dimensional electron gas (2DEG) formed by a GaAs/AlGaAs heterostructure. At small source-drain bias, we observe Coulomb oscillations when the gate voltages of the two dots are tuned so that both dots conduct, in agreement with classical models of the two-dot system. At higher source-drain bias these Coulomb oscillations develop sharp overstructure, which arises from the alignment and de-alignment of quantized energy levels of the two dots.

Considerable interest recently has been directed to the study of electron transport through double quantum dots [1–15]. Much attention already has been paid to metallic and metallic-like systems [1–5], where the average quantum level spacing $\Delta E$ is very fine ($\Delta E \ll kT$), making these systems well-suited to classical models. Coupled quantum dots with well-separated OD energy levels have also been studied theoretically [8–10] and experimentally [11–14]. Theories predict that the current through a double-dot system will be enhanced when the OD-levels of the dots are in alignment, and that the current will be suppressed when the levels are de-aligned. In this paper, we examine the effects of both charging and level quantization on electron transport through two dots connected in series.

Our device, fabricated from a GaAs/AlGaAs heterostructure, consists of an etched narrow channel of width 0.8 $\mu$m with five metal gates deposited over its width [15]. Two of the gates are held open at all times; the other three gates define the dots. Fig. 1a shows a diagram of the device. When negative voltages are applied to the three gates ($V_g, V_{g2}, V_{gm}$), the two-dimensional electron gas (2DEG) is depleted beneath the gates, forming two dots. The electron density was $3.6-4.0 \times 10^{11}$ electrons/cm$^2$ after illumination with an infrared diode at 4.2 K. The experiments were carried out in a dilution refrigerator, with the temperature of the electron gas determined to be 80–120 mK. All measurements were carried out near filling factor $\nu = 4$ ($B = 3.7-4.0$ T), where the level spectra of the dots are very regular. The dots were measured...
Fig. 1. (a) Schematic view of the double-dot device. Three gates $(V_{g1}$, $V_{g2}$, $V_{g3}$) define the dots and modify their charge state. The source–drain bias $V_{sd}$ is applied to the left lead, and the right lead is anchored to ground. (b) Theoretical phase diagram of a double-dot at low bias; current is only seen at the triple-points (denoted by solid circles). (c) Measurement of the current (plotted in gray-scale) through the double-dot at low source–drain bias as a function of the two gate voltages $V_{g1}$ and $V_{g2}$. Distinct triple-points can be observed at approximately $V_{g1} = 3.6$ mV, $V_{g2} = 2.2$ mV.

The charging of the dots individually and found to have a charging energy $e^2/C = 300–500 \ \mu$eV, and an average OD-state energy separation $\Delta E = 80–100 \ \mu$eV.

Before discussing our experimental results, we begin by considering the double-dot system using a classical model, where the effects of energy level quantization are ignored. It is useful to analyze this system by means of a “phase diagram” (Fig. 1b) [1], which details the charge–state of the dots as a function of the two gate voltages. The charge–state notation $(M/N)$ means that there are $M(N)$ electrons on dot 1(2). The phase diagram shown is for the case of the two gate capacitances being nearly equal, with vanishing cross-capacitance between one gate and the other dot. Beginning at any point in the diagram, sweeping $V_{g1}(V_{g2})$ positively will add electrons to dot 1(2) one by one. If the two dots are completely independent (meaning the interdot capacitance $C_{12} = 0$), the phase diagram will simply be an array of squares, with vertical (horizontal) lines corresponding to the location of Coulomb oscillations for dot 1(2). Transport occurs at low bias for two dots in series only when these lines intersect, so that charge fluctuations can occur for both dots. If the interdot capacitance $C_{12}$ between the dots is included, the charge–state domains become hexagonal, as the $(M,N)$ and $(M + 1,N + 1)$ charge–states are separated in energy due to electrostatic repulsion of electrons on different dots. At low bias, current can only flow through the two dots in series when the gate voltages are tuned to coincide with one of the triple-points (indicated by solid circles in Fig. 1b), where the three charge states involved in shuttling one electron across the double-dot system come together. For example, at the triple-point indicated by an arrow in the Figure, the double-dot system can fluctuate between charge–states $(M,N)$, $(M + 1,N)$, and $(M,N + 1)$. By applying a small bias, electrons are driven across the device as the system cycles from $(M,N)$ to $(M + 1,N)$ to $(M,N + 1)$ and back to $(M,N)$. At its neighbor triple-point, the sequence is $(M + 1,N + 1)$ to $(M + 1,N)$ to $(M,N + 1)$ and back to $(M + 1, N + 1)$, which shuttles a “hole” backwards across the device. Note that both triple-points involve a transition from $(M + 1,N)$ to $(M,N + 1)$, when an electron is moved from one dot to the other. We will return to this interdot transition later in this paper.

Fig. 1c shows a measurement of the current through our double-dot system as a function of $V_{g1}$ and $V_{g2}$, with the current plotted in grey-scale (black corresponds to zero current, white to $\sim 5$ pA). This roughly square array of Coulomb peaks demonstrates that we have two well-formed dots. The array is slightly skewed, due to a small cross-capacitance between gate voltages and their opposite dots. In addition, a pair of triple-points can be resolved in the upper right region of the plot. Nearly all the pairs of triple-points are blurred together because of a finite source–drain bias (50 $\mu$V) and thermal smearing, both of which limit resolution.

We now discuss the nonlinear properties of the two dots in the classical model. At high bias (in this case, negative bias), the regions of phase space where current is allowed grow from triple-points into triangular regions (Fig 2a). The size and orientation of these triangles can be accounted for...
by the condition that electrons, while moving through the two-dot system, never jump uphill in energy, i.e., $\mu_L \geq \mu_1 \geq \mu_2 \geq \mu_R$, (or $\mu_L \leq \mu_1 \leq \mu_2 \leq \mu_R$ for positive bias) where $\mu_L(R)$ is the potential of the left (right) lead and $\mu_{1(2)}$ is the potential of dot 1(2) when an extra electron is added to that dot. The three equations that must be satisfied $- \mu_L \geq \mu_1, \mu_1 \geq \mu_2$ and $\mu_2 \geq \mu_R$ - define the borders of the current-carrying triangular regions.

We repeat the type of measurement plotted in Fig. 1c, but now with a high source-drain bias ($-350 \mu V$). In Fig. 2b, the current through the two-dot system is plotted in grey-scale (black corresponds to zero current, white to $\sim 12 \mu A$). At high source-drain bias, we indeed see a square array of nearly-right-angled triangles. We also see extra features that cannot be accounted for by the classical model, namely a set of diagonal stripes moving up and to the right at a 45° angle; the point “X” is located on one of these stripes. Above and parallel to this primary stripe is a darker stripe, corresponding to a lower current, and further above is a second bright stripe which is less well-defined than the primary stripe. These stripes represent overstructure on each Coulomb oscillation. This overstructure is dramatically present in the line trace of Fig. 3 (taken during a different measurement run). Here the applied bias ($+380 \mu V$) is of the opposite sign to the bias used to generate Fig. 2b, but the explanation is similar. The plot shows four Coulomb oscillations with two or three sharp features per oscillation.

To understand this overstructure, we must consider the influence of the quantum levels on the transport step that involves tunneling between the two dots. Note that the stripes in Fig. 2b are all parallel to the $\mu_1 = \mu_2$ line shown in Fig. 2a, so that at every point along these stripes, $\mu_1 - \mu_2$ = const. This condition singles out the inter-dot tunneling step as being the origin of these features. Fig. 2c shows the potential landscape of the two-dot system at the point indicated by “X” in Fig. 2b. The quantum ground state for any particular charge state of dot 1(2) is labelled 0(0$'$) and the first few excited states are labelled 1, 2 (1', 2') [16]. The primary stripe of each triangle corresponds to the alignment of the ground-state energies of the two dots, $\mu_{1(0)} = \mu_{2(0')}$). If $V_{g2}$ is increased, the ground states of the dots are taken out of alignment, and the current drops (dark stripe). As we change $V_{g2}$ further $\mu_1(0)$ aligns with $\mu_2(1')$, and the current increases (second bright stripe). These stripes have no dependence on interdot charging.

![Fig. 2. (a) Theoretical phase diagram at high-bias ($0 < V_{sd} \leq e^2/C$). The hatched areas represent the gate voltages where current is observed, and the conditions for current flow define the borders of the hatched areas. (b) Gray-scale plot of the measured current through the double-dot system at high-bias ($V_{sd} = -0.35 \text{ mV}$). The diagonal stripes are due to the alignment of energy levels. (c) Potential landscape at the point “X” in Fig. 3b.](image)

![Fig. 3. Four double-dot Coulomb oscillations at high bias. The rightmost peak on each dot corresponds to the alignment of the ground-states of both dots (0-0'). The next peak corresponds to alignment of the excited state of dot 1 to the ground state of dot 2 (1-0').](image)
effects, because, as mentioned previously, the 
"electron" and "hole" sequences of current flow 
both contain the step, \((M+1,N)\) to \((M,N+1)\). It 
is the influence of OD-states on this transition rate 
that gives rise to the stripes in Fig. 2b.

The alignment of OD-states is responsible for 
the overstructure in the data of Fig. 3. The sharp 
peaks are due to alignment of the OD-states indi-
cated next to each peak; for example, 1-0' labels 
the peak in current which results from the first 
excited state of dot 1(1) aligning with the ground 
state of dot 2(0'). The spacing \(\Delta E\) between energy 
levels on dot 1 can be deduced from the spacing 
\(\Delta V_{ab}\) between the 0-0' and 1-0' resonances on any 
Coulomb oscillation. We obtain \(\Delta E \approx 90 \mu eV\), in 
agreement with single-dot measurements.

In conclusion, we have examined linear and non-
linear transport through two dots in series. We 
observed that the alignment of OD-states can 
dramatically affect transport through a double-dot 
system. Similar results to ours have been recently 
published by van der Vaart, et al. [14].

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[16] Fig. 3 shows the energy splittings for both dots being con-
stant and equal. This assumption is not necessary to 
explain the main features of this paper, it just simplifies 
the analysis. Nevertheless, our experiments indicate that 
the energy splittings of each dot are not appreciably 
different.