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Two coupled quantum dots with a continuous density of states

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We have studied tunnelling between two quantum dots with a continuous density of states. The Coulomb blockade oscillations show a double peak structure when the transmission between the dots is below unity. This peak-splitting is described in terms of the electrostatic interaction between the two dots using a classical capacitance description. This model neglects charge fluctuations due to tunnel events between the dots. We compare the temperature dependence of the split-peaks with a recent theory that does include charge fluctuations.

Key words: coulomb blockade, double quantum dots, tunneling.

1. Introduction

Single electron charging effects have been studied in detail in double metal tunnel junctions [1] and in semi-conductor quantum dots [2]. Transport through these nanostructures can be regulated by the Coulomb blockade of tunnelling. This leads to periodic Coulomb blockade oscillations as a function of the island potential. Multi-island structures have been studied by Pothier *et al*. [3] and Geerligs *et al*. [4] in the metallic regime, where the islands have a continuous density of states. It was found [4] that the Coulomb blockade oscillations split-up into double peaks for double island devices and into triple peaks for triple island devices. However, in these metal systems it is not possible to tune the tunnel barrier between the islands and to study the effect of tunnelling on the inter-island coupling.

More recently, multi-island structures were investigated using semi-conductor quantum dots. These studies showed a scaling of the charging energy with the area of the dot [5], a double periodicity due to the two charging energies of both dots [6], a stochastic suppression of the current through the double dot [7, 8] and an interaction between the dots [9–12]. The advantage of using these types of structures is the presence of tuneable tunnel barriers. Furthermore, small quantum dots can have a discrete energy spectrum [13], which

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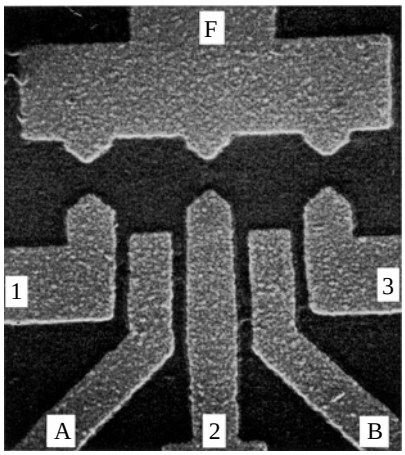

Fig. 1. SEM micrograph of the double dot geometry. Each dot has a lithographic size of 600×600 nm^2. This gate geometry allows a flexible control of the conductance of the three tunnel barriers 1-F, 2-F and 3-F. The coupling between the two dots can be tuned with voltage on gate 2.

allows the study of band structure effects in multi-dot structures with strongly coupled dots [14–17]. In the limit of weak coupling between the dots, sharp resonances were observed in the tunnel current when the energy of the two discrete states lined up [18, 19].

In this paper, we discuss experiments on a double quantum dot, where we can neglect the discrete energy spectrum of the dots. We study in detail the transition from two strongly coupled dots to two weakly coupled dots. As the coupling between the dots is increased, we observe a transition from equally spaced Coulomb oscillations to Coulomb oscillations with a double peak structure. A first understanding of this effect can be obtained in terms of a changing capacitance between the dots. However, a description with geometric capacitances in the strong tunnelling regime neglects the important role of charge fluctuations. We compare the temperature dependence of the conductance oscillations with a recent theory that includes quantum fluctuations due to inter-dot tunnelling [20, 21].

2. Device description and experiments

The double dot is defined by metal gates on top of a GaAs/AlGaAs heterostructure with a two-dimensional electron gas (2DEG) 100 nm below the surface. The ungated 2DEG has a mobility of 2.3×10^6 cm^2 Vs^{-1} and an electron density of 1.9×10^{15} m^{-2} at 4.2 K. A scanning-electron micrograph of the gate geometry is shown in Fig. 1. The lithographic size of each dot is 600×600 (nm)^2. Applying negative voltages to the six gates depletes the electron gas underneath them, and forms two quantum dots in the 2DEG. Current can flow from the large electron reservoir on the left via the three tunnel barriers induced by the gate pairs 1-F, 2-F and 3-F to the reservoir on the right. The coupling between the dots can be changed with the voltage on gate 2. The split-gate geometry allows us to characterize the individual dots and compare their properties to those of the double dot. Measurements at finite source–drain voltage were used to determine the charging energy E_A and E_B of the single dots ($E_A = E_B = 0.4$ meV). We did not observe any signatures of discrete energy states which indicates that the separation of the discrete states is small compared to temperature.

Figure 2B shows the conductance through the double dot versus the gate voltage V_{g2} using a d.c.-source–drain voltage of 30 μV. The conductance of QPC_1 and QPC_3 is fixed at a value well below $2e^2/h$. The data shows a transition of periodic oscillations into oscillations with a double peak structure. The double peaks are separated in gate voltage V_{g2} by Δ_2 and the groups of double peaks by Δ_1. The black dots in Fig. 2C denote the peak spacing ΔV_{g2} of the oscillations in this trace. Around $V_{g2} = -370$ mV, the peak spacing is roughly

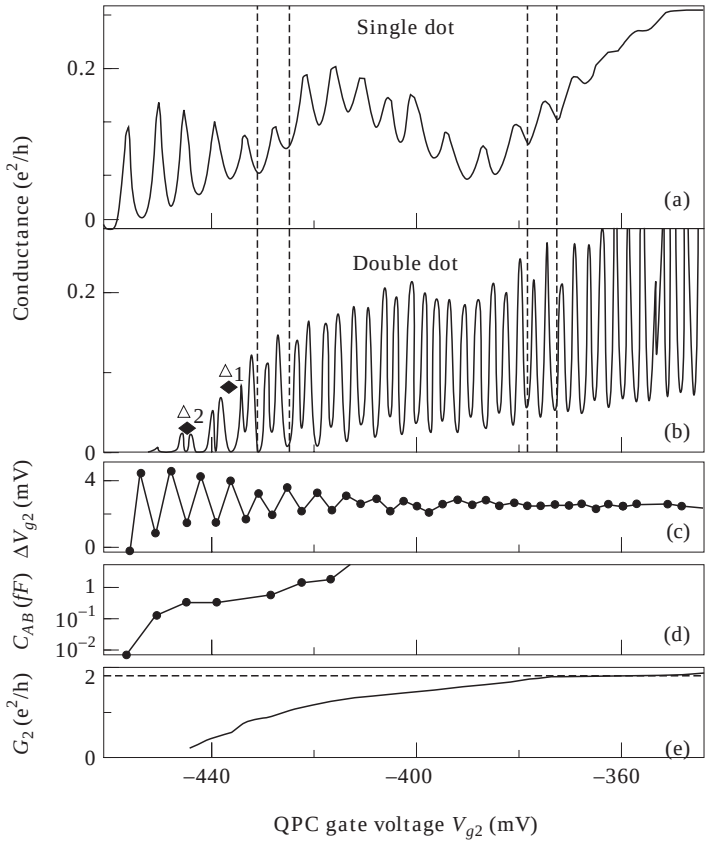

Fig. 2. (A) Current through dot A as a function of V_{g2} (B) Current through the double dot versus V_{g2}. (C) The black dots denote the peak-spacing of the oscillations in (B). (D) Calculated inter-dot capacitance C_{AB} from the data of (C) as a function of the gate voltage V_{g2}. (E) The conductance of QPC_2 versus V_{g2}. We have shifted this trace by 60 mV to account for the electrostatic influence of the other gates.

constant ($\Delta_1 = \Delta_2$). Upon decreasing the gate voltage V_{g2}, the peak spacing starts to show two different periodicities: Δ_2 decreases, while Δ_1 increases. Similar features were observed by Waugh *et al*. [9].

Figures 2A and 2B show a comparison between the conductance through a single dot (dot A) and the conductance through the double dot as a function of V_{g2}. The conductance through the single dot shows equidistant Coulomb oscillations. Around $V_{g2} = -380$ mV, the period of the Coulomb oscillations in the single dot is two times larger than the period $\Delta_1 = \Delta_2$ of the Coulomb oscillations in the double dot. As the gate voltage is decreased, each peak in the current through the single dot still corresponds to two peaks in the double dot. However, the peak spacing ΔV_{g2} of the double dot oscillations alternates, whereas the oscillations of the single dot remain periodic. Note that the peak spacing ΔV_{g2} in Fig. 2C starts to oscillate in the gate voltage region where the Coulomb oscillations in the single dot appears ($V_{g2} \approx -360$ mV in Fig. 2A). Around this gate voltage the conductance of QPC_2 drops below $2e^2/h$. This implies that the formation of the double peak structure in Fig. 2B starts when particle exchange between the dots occurs via tunnelling.

3. Electrostatic energy of a double quantum dot

The observed splitting of the conductance peaks in the double dot can be understood in terms of the electrostatic interactions between the dots. The total electrostatic energy $U(N_A, N_B)$ of a double dot with N_A

electrons in dot A and N_B electrons in dot B is given by [8]:

$$U(N_A, N_B) = \frac{1}{2}N_A^2 E_A + \frac{1}{2}N_B^2 E_B + N_A N_B E_{AB} + f(V_{g2}). \tag{1}$$

This formula shows that the energy of the double dot can be described with the charging energy E_A and E_B of two independent dots with an additional interaction term E_{AB}. The gate voltage dependence $f(V_{g2})$, and the charging energies can be expressed in terms of capacitances between the dots and their surroundings.

$$E_A = e^2 \frac{C_B + C_{AB}}{C_A C_B + C_{AB}(C_A + C_B)}, \qquad E_B = e^2 \frac{C_A + C_{AB}}{C_A C_B + C_{AB}(C_A + C_B)} \tag{2}$$

$$E_{AB} = e^2 \frac{C_{AB}}{C_A C_B + C_{AB}(C_A + C_B)} \tag{3}$$

$$f(V_g) = \frac{eC_{g2}V_{g2}}{C_A C_B + C_{AB}(C_A + C_B)}[N_A(C_B + 2C_{AB}) + N_B(C_A + 2C_{AB})]. \tag{4}$$

Here $C_A(C_B)$ is the total capacitance of dot A (dot B) due to the gates and the reservoirs and C_{AB} is the inter-dot capacitance.

When C_{AB} is much larger than all other capacitances, the inter-dot capacitance acts as a 'short'. In this strongly coupled limit, eqn (1) reduces to:

$$U(N_A, N_B) = \frac{e^2(N_A + N_B)^2}{2(C_A + C_B)} + e(N_A + N_B)\frac{2C_{g2}}{C_A + C_B}V_{g2} \tag{5}$$

i.e. the total electrostatic energy is the sum of the charging energy of a single dot containing $(N_A + N_B)$ electrons. In this regime, dot A and B have the same electro-chemical potential (ECP) and effectively form one 'large dot' ($V_{g2} > -400$ mV). The capacitance between this large dot and gate 2 is therefore two times larger than the capacitance C_{g2}. This corresponds to eqn (5), which describes a single dot with a capacitance $(C_A + C_B)$ and a gate capacitance of $2C_{g2}$. This is consistent with the experimental observations. The period of the Coulomb oscillations in the double dot is half of the period observed in the single dot (see Fig. 2A and B).

When C_{AB} is smaller than all other capacitances ($C_{AB} \to 0$), eqn (1) reduces to:

$$U(N_A, N_B) = \frac{(eN_A)^2}{2C_A} + eN_A\frac{C_{g2}}{C_A}V_{g2} + \frac{(eN_B)^2}{2C_B} + eN_B\frac{C_{g2}}{C_B}V_{g2} \tag{6}$$

i.e. the total electrostatic energy is the sum of the charging energy of two separate dots. This limit corresponds to the left-hand side of Fig. 2B, where the splitting has almost disappeared and the period of the oscillations is the same as in the single dot.

In the intermediate regime, when C_{AB} is of the same order as the other capacitances, charging one of the dots also affects the energy of the other dot. Adding one electron to dot A increases the ECP μ_A of dot A by E_A and μ_B by the interaction energy E_{AB}. Similarly, adding one electron to dot B increases μ_B by E_B and μ_A with the interaction energy E_{AB}. This gives rise to two transport mechanisms through the double dot [3]: $(N_A, N_B) \to (N_A, N_B - 1) \to (N_A - 1, N_B) \to (N_A, N_B) \to \cdots$ and $(N_A - 1, N_B) \to (N_A - 1N_B - 1) \to (N_A, N_B - 1) \to (N_A - 1, N_B) \to \cdots$. Each mechanism corresponds to one of the maxima of a split-peak in Fig. 2B. Each double peak corresponds to removing two electrons from the double dot (one electron from each dot). The splitting of the peaks (Δ_2) is proportional to the electrostatic interaction E_{AB} between the dots [3].

An estimation of E_{AB} can be made by considering the change in μ_A and μ_B as a function of V_{g2}. Using eqn (1) and $C_A = C_B = C$ gives:

$$\Delta\mu_A = \Delta\mu_B \propto e\frac{C_{g2}}{C}\Delta V_{g2}. \tag{7}$$

Using $C_{g2} = 3\times10^{-17}$ F, obtained from the period of the Coulomb oscillations and $C = 40\times10^{-17}$ F obtained

from the charging energy of the single dot, gives an estimate for the interaction energy: $E_{AB} = 150\ \mu$eV around $V_{g2} = -420$ mV, and decreases to 7 μeV around $V_{g2} = -460$ mV. This shows that the electrostatic interaction between the dots decreases when the coupling between the dots decreases. A simple interpretation of this effect is that tuning the gate voltage V_{g2} changes the height and the shape of the tunnel barrier between the dots. For decreasing gate voltage on the QPC 2, the distance between the two dots increases. This not only changes the transparency of the tunnel barrier, but also the capacitance C_{AB} between the dots.

To compare the coupling of the two dots to the inter-dot capacitance C_{AB}, we have plotted the conductance of QPC_2 and C_{AB} versus the gate voltage on gate 2 in Fig. 2D and E. Using eqns (3) and (7), we have calculated the inter-dot capacitance C_{AB} from the peak spacings Δ_2 in Fig. 2C. Figure 2D shows that the inter-dot capacitance C_{AB} diverges when the transmission of the barrier approaches unity. Note that despite this divergence, the total electrostatic energy of the double dot remains finite. This can be seen from the peak spacings: Δ_2 and Δ_1 both change in Fig. 2B as a function of gate voltage, the sum $(\Delta_2 + \Delta_1)$, however, is roughly constant (see Fig. 2C). This is in agreement with eqns 2 and 3. Whereas E_{AB}, E_A and E_B change as a function of C_{AB}, their sum is constant: $(E_{AB} + E_B) = (E_{AB} + E_A) = e^2/C \propto (\Delta_2 + \Delta_1)$, assuming again that $C_A = C_B = C$.

4. Temperature dependence of the peak amplitude

The above experiments are discussed in terms of a capacitance model, which neglects quantum fluctuations of the charge on the two dots. This assumption is unjustified when the inter-dot conductance approaches $2e^2/h$. The average number of electrons on each dot is then no longer quantized because the wave functions leak from one dot to the other. Recently, it was pointed out that tunnel processes between the dots can also cause a splitting of the conductance peaks [20, 21]. These theoretical works consider a double dot where the total charge $(N_A + N_B)$ cannot fluctuate, i.e. when the two barriers connecting the double dot to the reservoirs have a transmission well below unity. However, when the transmission *between* the dots approaches unity, the difference in the charge on the two dots $N_A - N_B$ starts to fluctuate. This reduces the total energy of the double dot and splits the conductance peaks [20, 21]. In this model, the splitting decreases when there are less quantum fluctuations, i.e. when the transmission between the two dots decreases, in agreement with our measurements.

Matveev *et al*. [20] predict a distinct temperature dependence of the maxima of the conductance peaks. In the weak coupling regime, the peak maxima $I_{\max}$ should be suppressed as $I_{\max} \sim T^{\gamma}$ with an exponent $\gamma = 5/4$ for identical dots. The suppression of the current is caused by a vanishing overlap between the two many-body ground states, before and after tunnelling, as the temperature is lowered [20].

We verified these predictions using a second sample of identical design. Figure 3A shows the current through this double dot as a function of the voltage on gate 2 using a source drain voltage of 15 μV. The measurement is performed at the base temperature of a dilution refrigerator (10 mK). The temperature of the electron gas, however, is typically around 100 mK as obtained from the line shape of the Coulomb oscillations in a single dot. Similar to Fig. 2B, a clear transition occurs from equally spaced peaks on the right-hand side to oscillations with a double peak structure around $V_{g2} = -325$ mV. On the left-hand side, in the weak coupling limit, the peaks become equally spaced, but now with a twice as large period as the oscillations on the right-hand side.

The maxima $I_{\max}$ of six different peaks versus temperature are plotted in Fig. 3B on a double logarithmic scale. The peaks, denoted by 1 and 2 are in the regime where the two dots form effectively one large dot. Peak 3 and 4 are in the strong coupling regime, while peak 5 and 6 are measured in the weak coupling regime (see Fig. 3A). The two solid lines show a fit to the data points in the temperature range 100–750 mK. In agreement with theory we observe a power law for peak 3 and 4. The value of the exponent differs only slightly from the expected value of $\gamma = 5/4$ for symmetrical dots. We find $\gamma = 1.2$ for peak 3 and $\gamma = 0.8$ for peak 4.

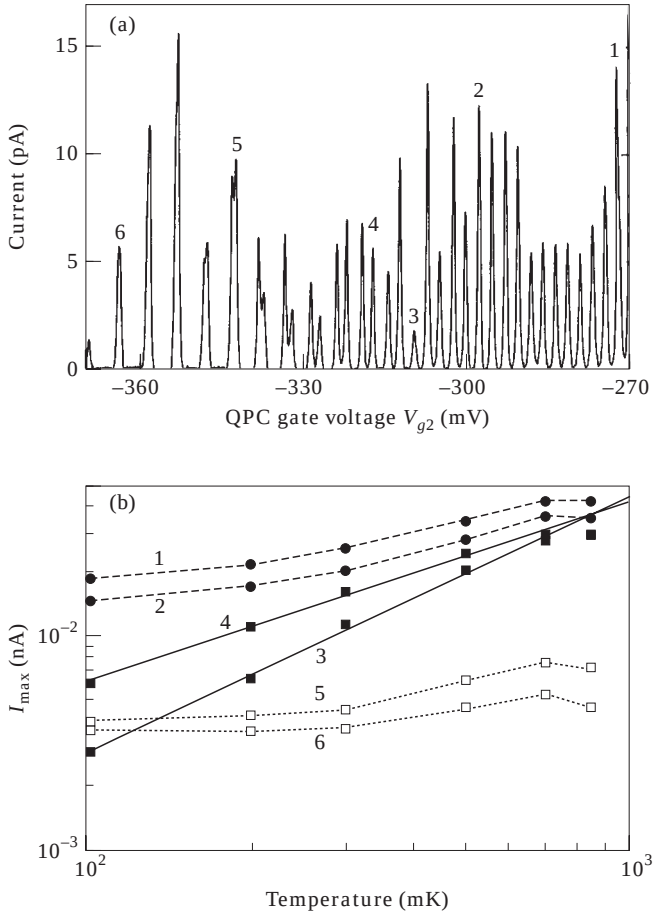

Fig. 3. (A) Conductance through the double dot as a function of the voltage on gate 2. (B) Temperature dependence of the peak maxima $G_{\max}$ in (A) denoted by 1 to 6.

A comparison between the six curves shows that the peak 1, 2 and 5, 6 do not exhibit a power law conforming to the orthodox Coulomb blockade theory. $I_{\max}$ for peak 5 and 6 (weak coupling regime) stays roughly constant for low temperatures and starts to increase around $T = 300$ mK. Similar features are seen in peak 1 and 2, where the double dot can be considered as one large dot. The distinct difference between those four peaks and the temperature dependence of the peaks in the strong coupling limit provides evidence that the tunnel mechanism proposed in Ref [20] describes the physics in the correct way.

5. Conclusions and discussion

We have shown that increasing the barrier between two quantum dots results in a transition of equally spaced oscillations to oscillations with a double peak structure. The transition occurs when the barrier conductance is $2e^2/h$ and the discrete character of the electron charge becomes important for moving charge between the dots. This effect can be described in terms of the electrostatic interactions between the dots using a variable inter-dot capacitance. As the barrier between the dots is decreased, the inter-dot capacitance increases and when the transmission probability between the dots approaches unity, the inter-dot capacitance goes to infinity. This

describes the physics of one 'large dot'. However, in this regime quantum fluctuations of the charge become important. A model which uses only geometric capacitances is then an oversimplification. Recently, these issues were considered theoretically for a single dot [22, 23]. The case of a double dot was discussed in Refs [20, 21]. Matveev *et al*. [20] predicted a power law for the peak maxima in the weakly coupled regime as a function of temperature. This is in agreement with our experiments. The exponent of the power law differs slightly from the theoretical prediction. An asymmetry in the two dots may be responsible for this deviation [20]. In addition to this effect, there can also be a deviation from the model due to a finite geometric inter-dot capacitance.

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